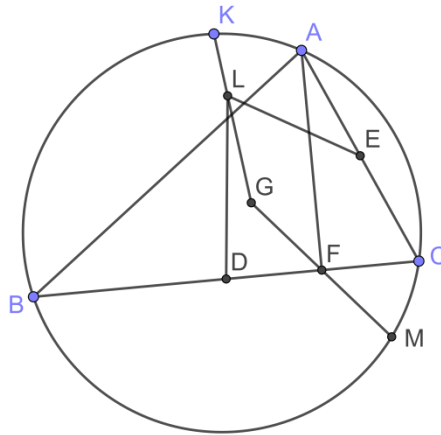


Cash Award Question of June-2025



In the picture, ΔABC is inscribed in the circle. D & E are the midpoints of BC & AC respectively. G is its Centroid and AF is altitude. K is a point on the minor arc AB and GK is joined. L is a point on GK such that $\angle ELD = \angle ACB$. GF is produced to meet the circle at M. Prove: KLFM is concyclic.

Question framed by
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Author's Solution June-2025

Given :

ΔABC is inscribed in the circle. D & E are the midpoints of BC & AC respectively. G is its Centroid and AF is altitude. K is a point on the minor arc AB and GK is joined. L is a point on GK such that $\angle ELD = \angle ACB$. GF is produced to meet the circle at M.

To Prove:

KLFM is concyclic.

Construction:

Draw the circumcircle of the ΔEFD .

Since E & D are the midpoints of AC & BC resp. and F is the foot of altitude, this circle is the TPC (Twelve Point Circle) of ΔABC . This TPC will pass through the midpoint of AB.

\Rightarrow J is the midpoint of AB. Join JD, JE & DE.

Proof:

J, D & E are the midpoints of the sides AB, BC & CA.

$$\Rightarrow \Delta JDE \sim \Delta CAB$$

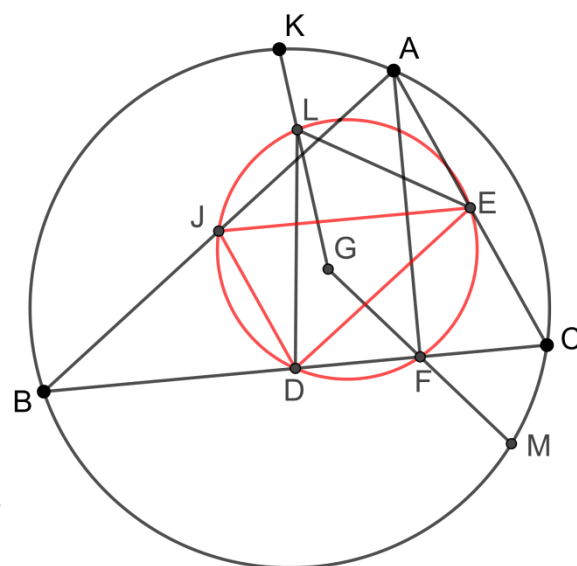
$$\Rightarrow \angle DJE = \angle ACB = \angle DLE \quad (\text{given})$$

\Rightarrow L lies on the TPC.

Now, G is centroid. GL & GF produced meet the main circle at M & K.

\therefore As per the "Centroid TPC Theorem VI" (page no: 21 of "**Advanced Theorems on Geometry**" available in this site), $GK \times GL = GM \times GF$.

\Rightarrow **KLFM is concyclic. ----- Proved.**



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